Now it’s time to look at…

Discrete Probability

(Chapter 5)

Discrete Probability

- Everything you have learned about counting constitutes the basis for computing the **probability** of events to happen.
- In the following, we will use the notion **experiment** for a procedure that yields one of a given set of possible outcomes.
- This set of possible outcomes is called the **sample space** of the experiment.
- An **event** is a subset of the sample space.

Discrete Probability

- If all outcomes in the sample space are equally likely, the following definition of probability applies:
- The probability of an event E, which is a subset of a finite sample space S of equally likely outcomes, is given by p(E) = |E|/|S|.
- Probability values range from **0** (for an event that will **never** happen) to **1** (for an event that will **always** happen whenever the experiment is carried out).

Discrete Probability

- **Example I:**
- An urn contains 4 blue balls and 5 red balls. What is the probability that a ball chosen at random from the urn is blue?
- **Solution:**
- There are 9 possible outcomes, and the event “blue ball is chosen” comprises four of these outcomes. Therefore, the probability of this event is 4/9 or approximately 44.44%.

Discrete Probability

- **Example II:**
- What is the probability of winning the lottery 6/49, that is, picking the correct set of six numbers out of 49?
- **Solution:**
- There are C(49, 6) possible outcomes. Only one of these outcomes will actually make us win the lottery.
- p(E) = 1/C(49, 6) = 1/13,983,816

Complimentary Events

- Let E be an event in a sample space S. The probability of an event –E, the **complimentary event** of E, is given by
- p(-E) = 1 – p(E).
- This can easily be shown:
- p(-E) = (|S| - |E|)/|S| = 1 - |E|/|S| = 1 – p(E).
- This rule is useful if it is easier to determine the probability of the complimentary event than the probability of the event itself.

Complimentary Events

- **Example I:** A sequence of 10 bits is randomly generated. What is the probability that at least one of these bits is zero?
- **Solution:** There are $2^{10} = 1024$ possible outcomes of generating such a sequence. The event –E, **"none of the bits is zero"**, includes only one of these outcomes, namely the sequence 1111111111.
- Therefore, p(-E) = 1/1024.
- Now p(E) can easily be computed as
$p(E) = 1 – p(-E) = 1 – 1/1024 = 1023/1024$.

Complimentary Events

- **Example II:** What is the probability that at least two out of 36 people have the same birthday?
- **Solution:** The sample space S encompasses all possibilities for the birthdays of the 36 people,
so $|S| = 365^{36}$.
- Let us consider the event –E ("no two people out of 36 have the same birthday"). –E includes P(365, 36) outcomes (365 possibilities for the first person's birthday, 364 for the second, and so on).
- Then $p(-E) = P(365, 36)/365^{36} = 0.168$,
so $p(E) = 0.832$ or 83.2%

Discrete Probability

- Let E_1 and E_2 be events in the sample space S. Then we have:

- $p(E_1 \cup E_2) = p(E_1) + p(E_2)$ **$- p(E_1 \cap E_2)$**

Does this remind you of something?

Of course, the principle of inclusion-exclusion.

Discrete Probability

- **Example:** What is the probability of a positive integer selected at random from the set of positive integers not exceeding 100 to be divisible by 2 or 5?

- **Solution:**
- E_2: "integer is divisible by 2"
E_5: "integer is divisible by 5"
- $E_2 = \{2, 4, 6, \ldots, 100\}$
- $|E_2| = 50$
- $p(E_2) = 0.5$

Discrete Probability

- $E_5 = \{5, 10, 15, \ldots, 100\}$
- $|E_5| = 20$
- $p(E_5) = 0.2$

- $E_2 \cap E_5 = \{10, 20, 30, \ldots, 100\}$
- $|E_2 \cap E_5| = 10$
- $p(E_2 \cap E_5) = 0.1$

- $p(E_2 \cup E_5) = p(E_2) + p(E_5) - p(E_2 \cap E_5)$
- $p(E_2 \cup E_5) = 0.5 + 0.2 - 0.1 = 0.6$

Discrete Probability

- What happens if the outcomes of an experiment are **not** equally likely?

- In that case, we assign a probability p(s) to each outcome $s \in S$, where S is the sample space.

- Two conditions have to be met:
- (1): $0 \leq p(s) \leq 1$ for each $s \in S$, and
- (2): $\sum_{s \in S} p(s) = 1$

- This means, as we already know, that (1) each probability must be a value between 0 and 1, and (2) the probabilities must add up to 1, because one of the outcomes is **guaranteed** to occur.